

Computational Models of Superconducting Quantum Effects

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Abstract—Through of computational and numerical models are created geometrical models of superconducting of type II, that is to say, Abrikosov superconducting to obtaining of a Fermionic prohibit energy trenches $\Delta(r)$, whose transition from a BCS - theory and through the application to any Majorana state $\psi(r)$, it meets that $\Delta(r) \cong \psi(r)$, where the BCS-theory has as limit GLAG + extensions of magnetic field, when $H \rightarrow H_c$, having valid the theory GLAG, to all temperature until critical temperature. Then by the Cooper pairs and under the spatial variation of the energy trenches and their magnetic images are developed these numerical models mentioned to their after study in superconducting of type II materials.

Keywords — Critical magnetic field, energy trenches, Fermionic superconducting region, fluxes, superconducting materials of type II.

I. INTRODUCTION

AN study of the microscopic effects of the superconductivity (BCS-theory [1]) in superconductors of type II, and some superconductors of type I save very special conditions, establish the existence of a differentiated energy regions (*energy bands or conduction bands, and trenches* [cite2]) studied in a metal whose Cooper pairs [1] can be formed by couplings that conduce us to them.

Other phenomenon that appears in the study of the superconductors of type II, are the Abrikosov vortices [3] that are intimately linked with the behavior of the magnetic field very near to the critical magnetic field H_c .

Finally to the respect of the vortices state, Abrikosov shows (and experimental is confirmed) that the structure of the vortices (filaments) is not arbitrary; these vortices are formed in the superconducting material drawing a triangular net. To the beginning, when is had the first vortices (we are near to H_c)

the separation between them is big, and if is followed the increase of the value of the applied magnetic field, each time penetrate more *fluxoids* [4, 5] (*are formed more vortices*) until that are so together that the distance between them is so only of the order of the coherent length ξ . Then we are in the value of the applied field equal to their second critical characterizing inside of the classification of the superconductors of type II, and that in this last point, the vortices collapse and the material return to the resistive normal state (superconducting zone or normal zone).

The determination and evaluation of superconducting materials near to the critical temperature T_c , establish that the major contribution of the energy of superconductivity comes of the Fermionic behavior of the corresponding Fock space [6] that can be determined around of the superconducting material provoking an energy trenches whose variation of energy, that can be controlled spatially by magnetism; result of the consideration of the trenches $\Delta(r)$, as a complex function that is annulled of a continuous way, non-sharp to T_c [7].

The theory of Gor'kov [8], using the technique of the Green functions re-interpret the mean of the BSC - Δ , parameter (prohibit energy trenches) doing possible the spatial variation mentioned.

The following research has as target to obtain many geometrical models of these re-interpreting of the BSC- Δ , and the formatting of their orbital spaces predicted by the microscopic theories having to limit the theory GLAG, to all temperature until critical temperature to superconductors of type II.

One of the phenomenons of quantum character that conform the appearing of orbital spaces in the superconducting phenomena is the tunnel effect, which is risked to differenced conditions between superconductor states densities and expressed as differential conductivity dI/dV .

In all it, the fermionic behavior is determinant to composite a sufficient Fermi liquid to conform a Fermi surface in the limit of the energy trenches.

The existence of an energy trenches in the superconductor state has the tunnel effect in a formed structure for superconductor-insulator-metal in normal state, has special

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characteristics [9]. It's clear that we need previously, to dispose of normal electrons for over of the superconducting trenches, that is to say, is necessary to break Cooper pairs as first option (*that is to say, minor applied potential differences than the trenches not will produce tunnel effect*). Then is had that potential differences applied to the barrier not produce tunnel current I , save, that defeat a threshold value that is precisely the bandwidth of trenches.

II. MEASUREMENTS AND SUPERCONDUCTING PARAMETERS

A problem that is required is the forming of strength coupling in the Cooper pairs, which requires of a phonons type that can be distinguishable through an extension of the BCS-theory that does distinguish the different contributions of the different phonons of the net of a superconducting material of type II.

If we could do a systematic study of the elements, alloys and superconducting composites seeing as response to the BSC-theory predictions, we would see that if the electron-phonon interaction is more intense (for example, are very highest the values of the resistivity in the normal state), major is the deviation in the theory. The extension of the BCS-theory to the situations in the electron-phonon coupling is strength, that are the majority of the superconductors, is one of the themes more complicated in the solid state physics. This theory was realized by Eliashberg and completed by Mc Millan [10, 11].

Assuming that the phononic coupling (coupling $\alpha(\omega) = \alpha = cte$) is constant near the Fermi energy is defined an average electron-phonon coupling as [11, 12]:

$$\alpha^2 F(\omega) = \frac{1}{N_{E_v}} \sum_{kk'} \sum_v |g_{k,k',v}|^2 \delta(\epsilon_{k'}) \delta(\epsilon_k) \delta(\omega - \omega_{qv}),$$

Theorem. (F. Bulnes). *Applied geometrical models to the strength electron-phonon interaction in the BCS-theory can be obtained for bounded variation using the spectral densities $\omega^2 \cos \omega$, or $\omega^3 \cos \omega$, in a superconductor of type II, in a superconducting state that can be extended to the Eliashberg-McMillan theory.*

The Feynman diagram corresponding to the interaction electron-phonon-electron has spectra in the purely BCS-theory, the density $\alpha^2 F(\omega)$, where $\alpha = \sqrt{|g_{k,k',v}|^2}$, is a coefficient that obeys to a Fermi energy of coupling between phonons and electrons to each mode in a superconducting metal of type II, in a superconducting state that can be extended to the Eliashberg-McMillan theory.

If the variations of $\alpha(\omega)$, have a behavior as ω^2 , or ω^3 , then these can be reflected in the interaction as a phonon link-wave (whose variation obeys to the theorem of bounded variation [13] considering the derivatives of $f(t) = \sin \omega_0 t^1$) in the corresponding Feynman diagram (see the figure 1). Indeed, if we consider as ordinary spectral electron-phonon density the function, for example, $\alpha^2 F(\omega) = \alpha^2 \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$, the function $F(\omega)$, has Fourier anti-transform $\cos \omega_0 t$, and $-\omega^3 \cos \omega$, is the 3th - order derivative of $f(t) = \sin \omega_0 t$.

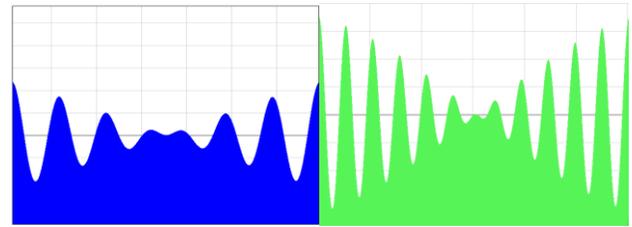


Figure 1. Spectra of electron-phonon-electron interaction. Observe that link-wave represented in the Feynman diagram (which a phonon wave) can be represented as pulse that increase at infinite in the two directions. This is explained for the excitation of electrons [7].

III. NUMERICAL MODELS AND COMPUTATIONAL WORK

Confirming the existing relations between the parameters of temperature near to the critical, the coherence parameter and the penetration longitude parameter in superconductors of type II, and their dependence with respect to the energy trenches, we can construct certain models that confirm the BCS-theory and their limit in the GLAG + extensions of magnetic field - theory. In this analysis, we will use the numerical models exalting the characteristics of magnitude in the values of superconducting energy always existing to the creating of energy trenches and their variation in the study and design of superconducting materials with specific properties to different applications.

Considerations in photonics and of superconducting microscopic effects establish that the step of modeling has that go given shape to single objects that after will be used in the scene to create the quantum effects required.

Likewise, as first model we obtain the energy surface $z = E(k_1, k_2)$, to prohibit energy trenches that produces a planar constant of momentary superconductivity [14]. To it we

¹ The function $f(t) = t^2 \sin(1/t)$, is of bounded variation on the interval $[0, 2/\pi]$.

consider the Hamiltonian H_z :

$$H_z = \sin(1 + \sigma_1) + \frac{3}{\sin(2 - \sigma_2)} + \sin(1 + \sigma_1) + \frac{10}{\sin(10 - \sigma_2)}, \quad (1)$$

The process of modeling can include some related activities with the preparation of the 3D-model to their measurement of others parameters in superconductivity (see the figure 2). In a range $x_{\min} = -5$, and $y_{\max} = 5$, in 3D.

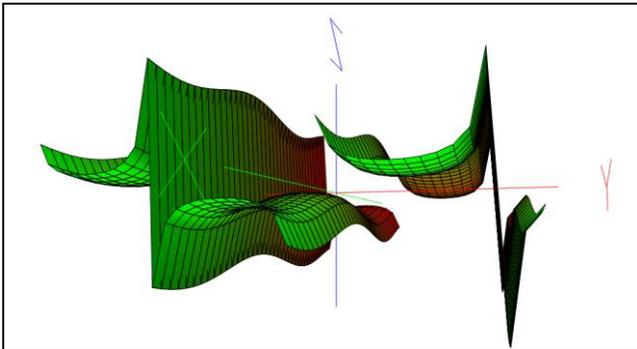


Figure 2. Prohibit energy trenches to superconductors. The energy surface $E(k_1, k_2)$, that produce a planar constant of momentary superconductivity [14]. The case that we see is with major generality, considering two different constants of superconductivity σ_1 , and σ_2 , (for example, to two metals to an alloy).

If we consider the critical temperature and the magnetic field corresponding to the temperature range $T_c > T$, and the limit

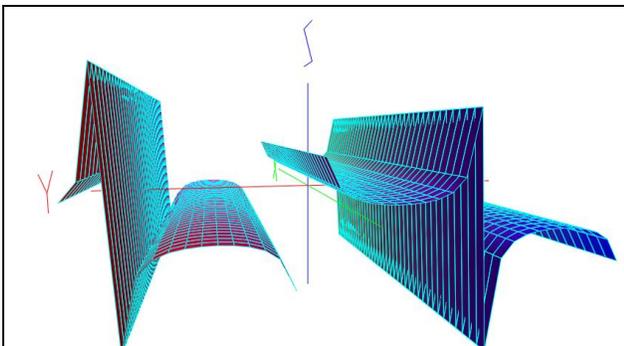


Figure 3. Representation of the energy trenches phenomenon considering the critical temperature of the magnetic field corresponding to the range $T \rightarrow T_c$, with $\sigma_1 = \sigma_2 = \sigma$.

$T \rightarrow T_c$, the free half walked of the electrons in the normal state (distance that walk between two consecutive interactions with the net) will be superior to the coherent length $\xi(T)$, which is known as clean superconductor (see the figure 3). Here the form of Hamiltonian is:

$$H_z = \sin(1 + \sigma) + \frac{10}{\sin(1 + \sigma)}, \quad (2)$$

The temperature condition establishes that inside a Cooper pair is had the intrinsic coherent longitude [4]:

$$\xi_0 = 0.18 \frac{\hbar v_F}{k_B T_c}, \quad (3)$$

Then the intrinsic coherent longitude (*which is a intrinsic material parameter*) depends only of the critical temperature and whose physical mean is the size of the Cooper pairs, while that the coherent longitude due to Ginzburg-Landau $\xi(T)$, [10, 11] depends of temperature T , that is to say, conforms the zone in that is had the spatial variation of the trenches $\Delta(r)$. The size of the Cooper pairs can be put in a limit condition on the finite length that must comply of difference temperatures in the border “before and after” of the trenches (considering the flow of electrons and the establishment of phonons predicted in the figure 1):

$$|T(\sigma) - T(-\sigma)| < \infty, \quad (4)$$

which is good condition to our measures as given in (3).

The computational work in the figures 2 and 3, require that model let designed in the space applying mathematical compute combining 2D and 3D-models to create enriched controls of the fine measures doing vary the superconductivity, the temperature and Fermi surface parameters.

An expanded calculus to Cooper pairs in the Fermion interaction can be interpreted as photon field where the trenches is evidenced (Figure 4).

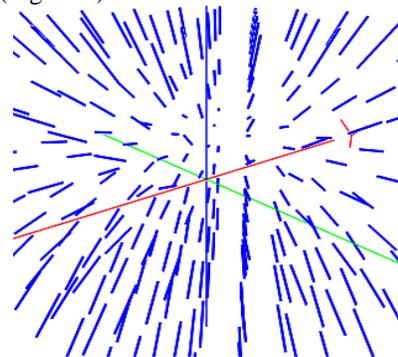


Figure 4. Fermionic Fock space corresponding that can be determined around of the superconducting material provoking an energy trenches whose variation of energy, that can be controlled spatially by magnetism.

The 3D-model of the figure 4, can some activities of the orbit spaces forming and related with the preparing of the 3D-model to their after animation. The following animation in the next paper will be in a simulating ANSYS or COMSOL [15].

The Eliashberg-McMillan theory is enough complicated as to that one expression of the critical superconducting

temperature T_c , in function of the spectral parameter $\alpha^2 F(\omega)$, be very little manageable. Now, when is had that $\lambda < 2$, (come from the relation between strength coupling and the corrections of the energy of the quasi-particles due to purely Coulumbian interactions (fermions showed in the figure 4)²).

Then can be use an approximated expression known as McMillan equation:

$$T_c = \frac{\theta_D}{1.45} \exp \left[- \frac{1 - 0.4(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)} \right], \quad (5)$$

where θ_D , is the Debye temperature and the rest of the parameters have the significant since acquaintance in the microscopic theory of the superconductivity.

Using the functional derivative of the critical temperature given by $\frac{\delta T_c}{\delta \alpha^2 F(\omega)}$, we compute to the critical temperature

T_c , to a value of $\alpha^2 F(\omega_0)$, and then is calculated a new critical temperature to the value of modified $\alpha^2 F(\omega)$, with certain weight, to certain near frequency to the before. That is to say, are used the little differences of the bounded variations suggested by the theorem and the existing statistical weight in the Feynman diagram corresponding to the figure 1, stay intimately related with the weight of said frequency (see figure 5).

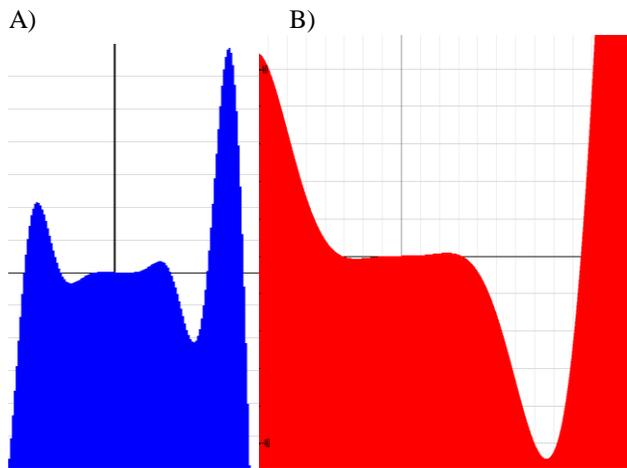


Figure 5. A) Electron-phonon spectral density models

²Equation $1 + \lambda = \frac{m^*}{m}$, where the parameter λ , carry on itself the

idea of the “clothing” electron (means that the electron in their movement polarize the net and is accompanied for a phonons cloud, for example, to the metal **Pb**, their value is $\lambda = 1.55$).

$\alpha^2 F(\omega) = -\omega^3 \cos \omega$, or $\omega^3 \cos \omega$, used to prove their necessity of bounded variation in certain finite interval. B) Spectral density of electron-phonon interaction for the intensity of the electron-phonon coupling obtained for tunnel effect in metal alloys superconductors [4].

IV. APPLICATIONS

Through the relation between a magnetic field near to H_c , and the range of temperature tends to the critical temperature we can have the behavior of an electro-twistor generated by the magnetic field-superconducting interaction to levitation [7] (see the figure 6).

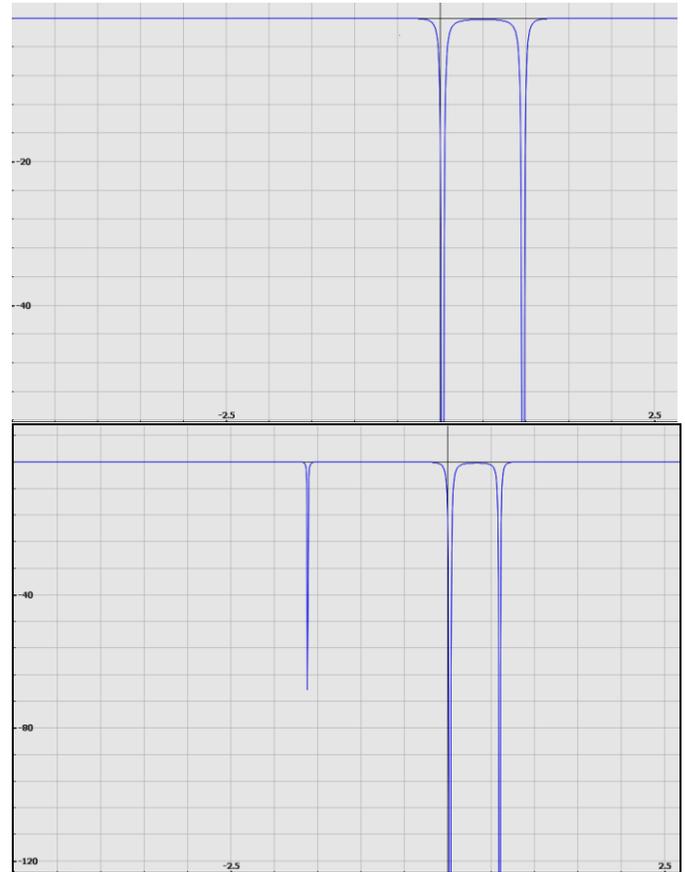


Figure 6. Electro-anti-gravitational effect having present the process of superconductor magnetic field that produces the Abrikosov effect.

At extremely low temperatures an electron can draw the positive ions in a superconducting material towards it. The movement of the ions creates a more positive region that attracts another electron to the area (see the figure 7).

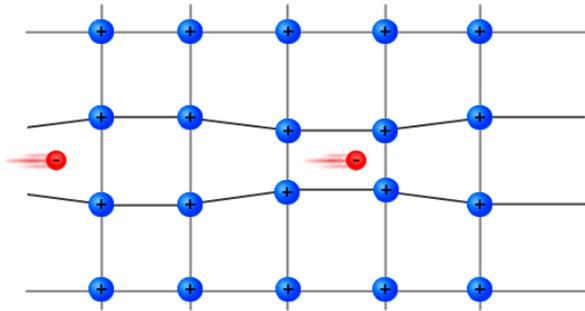


Figure 7. Cooper pairs make superconductors in their crystalline net.

Finally, and doing use of the computational modeling of the spatial variation of the energy trenches we can obtain a final representation considering other effects due to the matter condensation phenomena (see the figure 8) through of fog created by the thermal conditions obtained around of the magnetic field-superconducting interaction (and other whose origin let be Fock space created [6]) in the electro-anti-gravitational effect to magnetic levitation (figure 6).

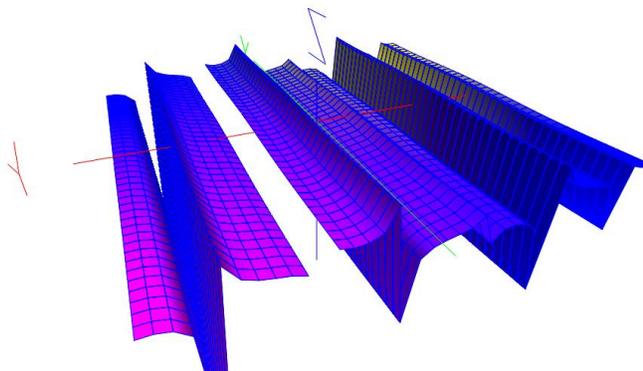


Figure 8. Energy trenches with Hamiltonian $H = \sin(1 + \sigma_1) + 3/\sin(2 - \sigma_2) + \sin(1 + \sigma_1) + 10/\sin(10 - \sigma_2)$.

The graphics for hardware use the *Goround shaped*, which interpolates the colors to along of the surface starting of the compute of the illumining in the vertex of the primitive obtaining that the object has a more realistic aspect (see the figure 8).

V. CONCLUSIONS

The creating of geometrical models obeys to one correct interpretation of the prohibit energy trenches forming inside of the range of critical temperature and the tendency of the magnetic field to a critical magnetic field in the case of the superconductors of type II. But due the asymmetric inconsistencies in the conceptual map of the superconducting theory, arisen between their different microscopic theories as the GLAG, BCS, and Eliashberg-McMillan theories, are needed models that permit establish the variations of some parameters and their different behavior under relations between them. One of these relations are, for example, the critical

magnetic in their two phases (passing from the superconductors of type I to typeII), and the forming of the energy trenches due to the fermionic interactions establishing two orbital spaces good differentiated by the energy trenches, which are achieved with relations between of the coherent length parameter $\xi(T)$, in function of the temperature in the range $T < T_c$, the penetrating length $\lambda(T)$, and their Ginzburg-Landau parameter k , not being less important the interaction electron-phonon which in reality mark the fundamental process to achieve the fundamental property of the superconducting material, if this is designed in their crystalline net as a superconductor of type II, with a extended theory as **GLAG + extensions of magnetic field**, and whose variations in the couplings electron-phonon can be bounded and thus measured through the spectral density normed in $\alpha^2 F(\omega)$. In this case, the interval of definition must be extremely short, and this could represent a range of control via Fourier spectra of the achievement of the couplings and interactions electron-phonon in the search of the adequate alloy [16] with a temperatures ambient range.

VI. ACKNOWLEDGMENTS

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